# Heat Problems

specific heat of copper = $3.85 \times 10^2$ J kg$^{-1}$ K$^{-1}$
specific heat of steel = $4.50 \times 10^2$ J kg$^{-1}$ K$^{-1}$
specific heat of aluminium = $8.80 \times 10^2$ J kg$^{-1}$ K$^{-1}$
specific heat of iron = $4.77 \times 10^2$ J kg$^{-1}$ K$^{-1}$
*Other constants can be found in your Formulae and Data Booklet.*

1. 0.1 kg of an unknown metal is found to require 3.5 kJ to change its temperature from 25 °C to 82 °C. What is the specific heat of the metal?

**Solution:**

$$Q = mc\Delta T$$
$$c = \frac{Q}{m\Delta T}$$
$$c = \frac{3500}{0.1 \times 57}$$
$$c = 6.14 \times 10^2 \text{ J kg}^{-1}\text{ K}^{-1}$$

2. A piece of copper has $1.74 \times 10^4$ J of energy added to it to change its temperature from 20 °C to 80 °C. What was the mass of copper?

**Solution:**

$$Q = mc\Delta T$$
$$m = \frac{Q}{c\Delta T}$$
$$m = \frac{1.74 \times 10^4}{385 \times 60}$$
$$m = 0.753 \text{ kg}$$

3. If 15.7 kJ of heat energy is added to 250 mL of water at 20 °C, what will the water's new temperature be?

**Solution:**

$$Q = mc\Delta T$$
$$\Delta T = \frac{Q}{mc}$$
$$T_f - 20 = \frac{15.7 \times 10^3}{0.25 \times 4180}$$
$$T_f = 15.024 + 20$$
$$T_f = 35.0 \text{ °C}$$

4. Over a period of 6 hours, a hot water bottle cools from 95 °C to 20 °C. If it held 2.5 L of water, what was its rate of cooling in J s$^{-1}$?

**Solution:**

$$Q = mc\Delta T$$
$$Q = 2.5 \times 4180 \times 75$$
$$Q = 783750 \text{ J}$$
$$\text{energy per second} = \frac{783750}{6 \times 60 \times 60}$$
$$\text{energy per second} = 36.3 \text{ J s}^{-1}$$

5. A kettle rated at 2000 W contains 1.8 L water at 15 °C. If it runs for 3.5 minutes, will the water boil?

**Solution:**

$$P = \frac{Q}{t}$$
$$Q = Pt$$
$$Q = 2000 \times 3.5 \times 60$$
$$Q = 420000 \text{ J}$$

$$Q = mc\Delta T$$
$$420000 = 1.8 \times 4180 \times (T_f - 15)$$
$$T_f - 15 = \frac{420000}{1.8 \times 4180}$$
$$T_f - 15 = 55.82$$
$$T_f = 55.82 + 15$$
$$T_f = 71.8\ ^\circ\text{C}$$

Therefore the water will not boil.

6. How much heat energy is released when 423 g of steam at 100 °C condenses to water also at 100 °C?

**Solution:**

$$Q = mL$$
$$Q = 0.423 \times 2.26 \times 10^6$$
$$Q = 9.56 \times 10^5 \text{ J}$$

7. $4.87 \times 10^5$ J of heat are added to a mass of ice at 0 °C. If the ice melts and becomes water at 21.5 °C, what was its mass?

**Solution:**

$$Q_T = Q_{\text{melting ice}} + Q_{\text{heating water}}$$
$$Q_T = mL_f + mc\Delta T$$
$$4.87 \times 10^5 = (m \times 3.34 \times 10^5) + (m \times 4180 \times 21.5)$$
$$4.87 \times 10^5 = 3.34 \times 105m + 89870m$$
$$4.87 \times 10^5 = 423870m$$
$$m = 1.15 \text{ kg}$$

8. At what rate in J s$^{-1}$ is a freezer absorbing heat if 2.15 kg of water at 21.5 °C is just frozen in 2.0 hours?

**Solution:**

$$Q_T = Q_{\text{cooling water}} + Q_{\text{freezing water}}$$
$$= mc\Delta T + mL_f$$
$$= (2.15 \times 4180 \times 21.5) + (2.15 \times 3.34 \times 10^5)$$
$$= 911320.5 \text{ J}$$
$$\text{rate} = \frac{911320.5}{(2 \times 60 \times 60)}$$
$$= 127 \text{ J s}^{-1}$$

9. 20 g of milk at 5.0 °C is added to 250 g of coffee at 90 °C. What is the final temperature of the drink? (Specific heats: milk: $3.9 \times 10^3$ J kg$^{-1}$ K$^{-1}$, coffee: $4.10 \times 10^3$ J kg$^{-1}$ K$^{-1}$)

**Solution:**

$$\text{heat lost by coffee} = \text{heat gained by milk}$$
$$(mc\Delta T)_{\text{coffee}} = (mc\Delta T)_{\text{milk}}$$
$$0.25 \times 4100 \times (90 - T_f) = 0.02 \times 3900 \times (T_f - 5)$$
$$92250 - 1025T_f = 78T_f - 390$$
$$92640 = 1103T_f$$
$$T_f = \frac{92640}{1103}$$
$$T_f = 84.0\ ^\circ\text{C}$$

10. 100 g of a metal at 95 °C is added to 500 mL of water at 2.0 °C. If the final temperature of the water is 3.6 °C, what is the specific heat of the metal?

**Solution:**

$$\text{heat lost by metal} = \text{heat gained by water}$$
$$(mc\Delta T)_{\text{metal}} = (mc\Delta T)_{\text{milk}}$$
$$0.1 \times c \times (95 - 3.6) = 0.5 \times 4180 \times (3.6 - 2)$$
$$c = \frac{3344}{9.14}$$
$$c = 366 \text{ J kg}^{-1} \text{ K}^{-1}$$

11. How much heat energy is needed to change 1.0 kg of ice at -3.0 °C to steam at 107 °C?

**Solution:**

$$Q_T = Q_{\text{heating ice}} + Q_{\text{melting ice}} + Q_{\text{heating water}} + Q_{\text{boiling water}} + Q_{\text{heating steam}}$$
$$= (mc\Delta T)_{\text{ice}} + mL_f + (mc\Delta T)_{\text{water}} + mL_v + (mc\Delta T)_{\text{steam}}$$
$$= (1 \times 2100 \times 3) + (1 \times 3.34 \times 10^5) + (1 \times 4180 \times 100) + (1 \times 2.26 \times 10^6) + (1 \times 2000 \times 7)$$
$$= 3032300$$
$$Q = 3.03 \times 10^6 \text{ J}$$

12. How much ice at 0 °C must be added to 250 mL of coffee (specific heat: $4.10 \times 10^3$ J kg$^{-1}$ K$^{-1}$) in an insulated cup to cool the coffee from 95 °C to 65 °C? Assume that there is no loss of heat to the container and surroundings.

**Solution:**

$$\text{heat lost by coffee} = \text{heat gained by ice}$$
$$(mc\Delta T)_{\text{coffee}} = mL_f + (mc\Delta T)_{\text{water}}$$
$$0.25 \times 4100 \times (95 - 65) = (m \times 3.34 \times 10^5) + (m \times 4180 \times 65)$$
$$30750 = 605700m$$
$$m = \frac{30750}{605700}$$
$$m = 5.08 \times 10^{-2} \text{ J}$$

13. Copper calorimeters are used to determine the specific heat of unknown substances. A calorimeter of mass 41 g has 100 mL of water at 15 °C placed in it. 50 g of iron is heated to 160 °C then carefully lowered into the water. What would be the final temperature of the water?

**Solution:**

$$Q_{\text{iron}} = Q_{\text{water}} + Q_{\text{copper}}$$
$$(mc\Delta T)_{\text{iron}} = (mc\Delta T)_{\text{water}} + (mc\Delta T)_{\text{copper}}$$
$$0.05 \times 477 \times (160 - T_f) = (0.1 \times 4180 \times (T_f - 15)) + (0.041 \times 385 \times (T_f - 15))$$
$$3816 - 23.85T_f = 418T_f - 6270 + 15.785T_f - 236.775$$
$$10322.775 = 457.635T_f$$
$$T_f = 22.6 \text{ °C}$$

14. 5.0 g of ice at -2.0 °C is placed into a 78 g copper calorimeter containing 120 mL of water at 90 °C. The water is stirred until all the ice has just melted. What is the final temperature of the water?

**Solution:**

$$(mc\Delta T)_{\text{water}} + (mc\Delta T)_{\text{copper}} = (mc\Delta T)_{\text{ice}} + mL_{\text{f}} + (mc\Delta T)_{\text{cold water}}$$

$$(0.12 \times 4180 \times (90 - T_{\text{f}})) + (0.078 \times 385 \times (90 - T_{\text{f}})) =$$

$$(0.005 \times 2100 \times 2) + (0.005 \times 3.34 \times 10^5) + (0.005 \times 4180 \times T_{\text{f}})$$

$$45144 - 501.6T_{\text{f}} + 2702.7 - 30.03T_{\text{f}} = 21 + 1670 + 20.9T_{\text{f}}$$

$$46155.7 = 552.53T_{\text{f}}$$

$$T_{\text{f}} = \frac{46155.7}{552.53}$$

$$= 83.5\ °\text{C}$$

15. A 5.45 kg steel container contains 12.0 kg of water at 22.0 °C. When 2.65 kg of molten alloy (latent heat of fusion $2.5 \times 10^4$ J kg$^{-1}$ K$^{-1}$) at its melting point of 327 °C is poured into the water, the final temperature reached is 27.8 °C. Find the specific heat of the alloy.

**Solution:**

$$(mL_{\text{f}})_{\text{alloy}} + (mc\Delta T)_{\text{alloy}} = (mc\Delta T)_{\text{steel}} + (mc\Delta T)_{\text{water}}$$

$$(2.65 \times 2.5 \times 10^4) + (2.65 \times c \times 299.2) = (5.45 \times 450 \times 5.8) + (12 \times 4180 \times 5.8)$$

$$66250 + 792.88c = 14224.5 + 290928$$

$$c = \frac{238902.5}{792.88}$$

$$= 301\ \text{J kg}^{-1}\ \text{K}^{-1}$$

16. How much ice at -4.00 °C must be added to an aluminium calorimeter of mass 47.0 g containing 150 g of water at 95.0 °C so that the final temperature is 70 °C?

**Solution:**

$$(mc\Delta T)_{\text{aluminium}} + (mc\Delta T)_{\text{water}} = (mc\Delta T)_{\text{ice}} + (mL_{\text{f}})_{\text{ice}} + (mc\Delta T)_{\text{cold water}}$$

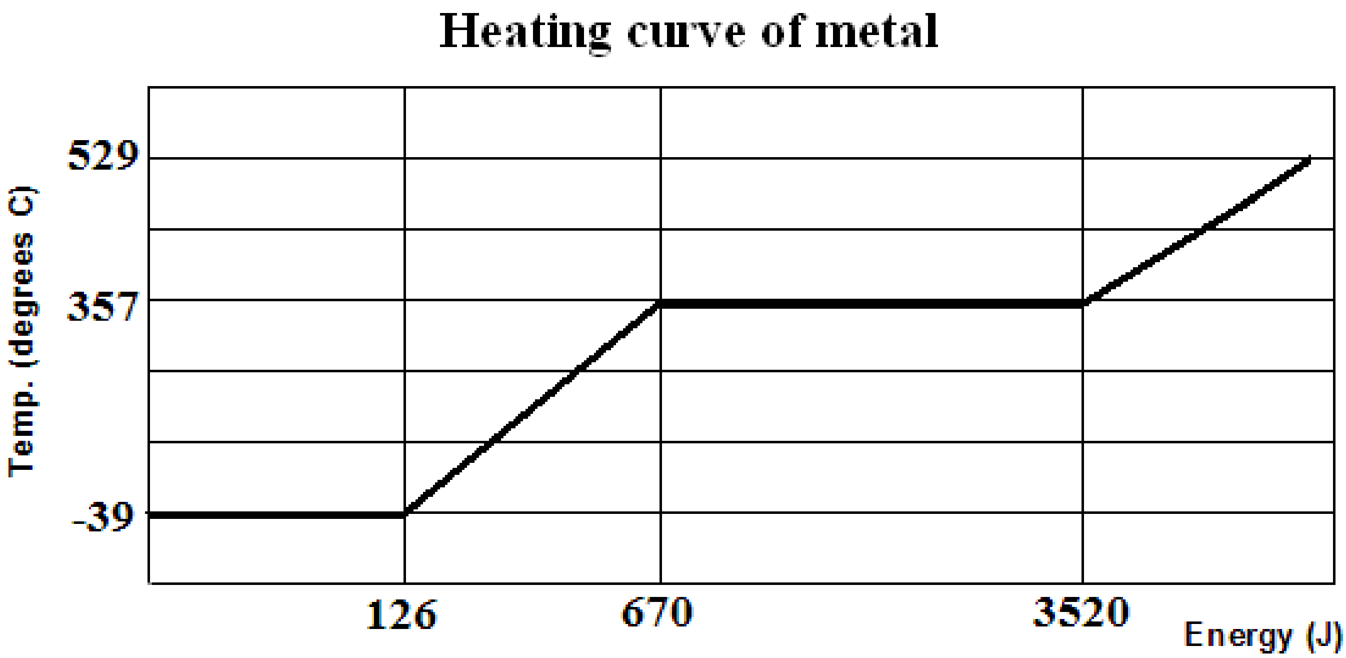
$$= m((c\Delta T)_{\text{ice}} + L_{\text{f ice}} + (c\Delta T)_{\text{cold water}})$$

$$(0.047 \times 880 \times 25) + (0.15 \times 4180 \times 25) = m(2100 \times 4 + 3.34 \times 10^5 + 4180 \times 70)$$

$$16709 = 635000m$$

$$m = 2.63 \times 10^{-2}\ \text{kg}$$

17. The graph below represents the heating curve for a metal. Energy is added to 10.0 g of the solid metal, initially at a temperature of -39 °C, until the metal evaporates (*graph not to scale*).




What is the specific heat of the metal in its liquid state?

**Solution:** Liquid state is between 126 and 670 J (-39 and 357 °C), so:

$$Q = 670 - 126 = 544 \text{ J}$$
$$\Delta T = 357 - (-39) = 396 \text{ °C}$$

$$Q = mc\Delta T$$
$$544 = 0.01 \times c \times 396$$
$$c = \frac{544}{0.01 \times 396}$$
$$= 137 \text{ J kg}^{-1} \text{ K}^{-1}$$